





a)  $231 \text{ cm}^2$

b)  $250 \text{ cm}^2$

c)  $220 \text{ cm}^2$

d)  $200 \text{ cm}^2$

15. Cards marked with numbers 1, 2, 3, ..., 25 are placed in a box and mixed thoroughly and one card is drawn at random from the box. The probability that the number on the card is a multiple of 3 or 5 is [1]

a)  $\frac{8}{25}$

b)  $\frac{12}{25}$

c)  $\frac{4}{25}$

d)  $\frac{1}{5}$

16. For a symmetrical frequency distribution, we have [1]

a) mean &lt; mode &lt; median

b) mode =  $\frac{1}{2}$ (mean + median)

c) mean = mode = median

d) mean &gt; mode &gt; median

17. The maximum volume of a cone that can be carved out of a solid hemisphere of radius 'r' is [1]

a)  $\pi r^3$

b)  $\frac{2}{3}\pi r^3$

c)  $\frac{1}{3}\pi r^3$

d)  $\frac{1}{3}\pi r^2 h$

18. If  $u_i = \frac{x_i - 25}{10}$ ,  $\sum f_i u_i = 20$ ,  $\sum f_i = 100$ , then  $\bar{x} =$  [1]

a) 24

b) 25

c) 23

d) 27

19. **Assertion (A):** Distance of point (a, b) from origin is  $\sqrt{b^2 - a^2}$  [1]

**Reason (R):** Distance of point (x, y) from origin is  $\sqrt{(x - 0)^2 + (y - 0)^2}$

a) Both A and R are true and R is the correct explanation of A.

b) Both A and R are true but R is not the correct explanation of A.

c) A is true but R is false.

d) A is false but R is true.

20. **Assertion (A):** The HCF of two numbers is 18 and their product is 3072. Then their LCM = 169. [1]

**Reason (R):** If a, b are two positive integers, then  $\text{HCF} \times \text{LCM} = a \times b$ .

a) Both A and R are true and R is the correct explanation of A.

b) Both A and R are true but R is not the correct explanation of A.

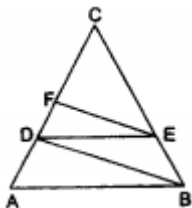
c) A is true but R is false.

d) A is false but R is true.

### Section B

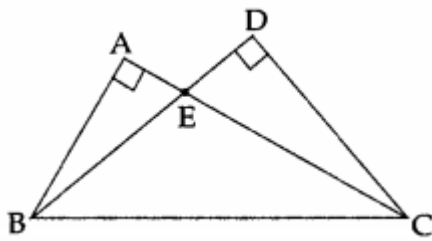
21. On comparing the ratios  $\frac{a_1}{a_2}$ ,  $\frac{b_1}{b_2}$  and  $\frac{c_1}{c_2}$ , find out whether the lines representing the pair of linear equations intersect at a point, are parallel or coincident:  $5x - 4y - 8 = 0$ ;  $7x + 6y - 9 = 0$ . [2]

22. In the given figure,  $AB \parallel DE$  and  $BD \parallel EF$ . Prove that  $DC^2 = CF \times AC$ . [2]



OR

In the adjoining figure, two triangles ABC and DBC are on the same base BC in which  $\angle A = \angle D = 90^\circ$ . If CA and BD meet each other at E, show that  $AE \times CE = BE \times DE$ .



23. Prove that the tangents drawn at the ends of a diameter of a circle are parallel. [2]
24. If  $\tan A = 1$  and  $\sin B = \frac{1}{\sqrt{2}}$ , find the value of  $\cos(A+B)$  where A and B are both acute angles. [2]
25. Find the area of the segment of a circle of radius 14 cm, if the length of the corresponding arc APB is 22 cm. [2]

OR

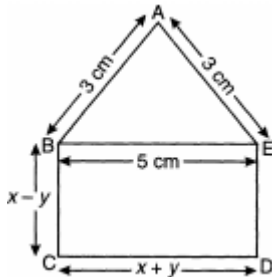
A chord 10 cm long is drawn in a circle whose radius is  $5\sqrt{2}$  cm. Find the areas of both the segments. [Take  $\pi = 3.14$ .]

### Section C

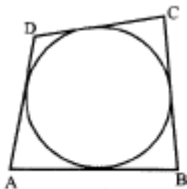
26. Show that  $5 - \sqrt{3}$  is irrational. [3]
27. Find the zeros of  $f(x) = x^2 - 2x - 8$  and verify the relationship between the zeros and its coefficients. [3]
28. The sum of the digits of a two-digit number is 9. Also, nine times this number is twice the number obtained by reversing the order of the number. Find the number. Solve the pair of the linear equation obtained by the elimination method. [3]

OR

In the figure below ABCDE is a pentagon with  $BE \parallel CD$  and  $BC \parallel DE$ . BC is perpendicular to CD. If the perimeter of ABCDE is 21 cm, find the Values of x and y.



29. In the adjoining figure, a quadrilateral ABCD is drawn to circumscribe a circle, Prove that  $AB + CD = AD + BC$  [3]



30. Prove that  $\frac{\sin \theta - \cos \theta + 1}{\sin \theta + \cos \theta - 1} = \frac{1}{\sec \theta - \tan \theta}$ , using identity  $\sec^2 \theta = 1 + \tan^2 \theta$ . [3]

OR

If  $\sin \theta + \cos \theta = \sqrt{2}$ , then evaluate  $\tan \theta + \cot \theta$ .

31. What is the probability that a randomly taken leap year has 52 Sundays? [3]

### Section D

32. The length of the sides forming right angle of a right triangle are  $5x$  cm and  $(3x - 1)$  cm. If the area of the triangle is  $60 \text{ cm}^2$ . Find its hypotenuse. [5]

OR

A cottage industry produces a certain number of toys in a day. The cost of production of each toy (in rupees) was found to be 55 minus the number of toys produced in a day. On a particular day, the total cost of production was ₹

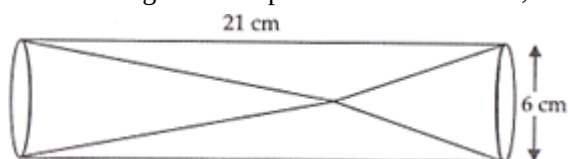
750. We would like to find out the number of toys produced on that day. Represent the situations mathematically (quadratic equation).

33. Sides AB and AC and median AD of a triangle ABC are respectively proportional to sides PQ and PR and median PM of another triangle PQR. Show that  $\Delta ABC \sim \Delta PQR$ . [5]

34. A toy is in the form of a cone mounted on a hemisphere. The diameter of the base of the cone is 7 cm and its height is 15.5 cm. Find the volume of the toy. (Use  $\pi = 3.14$ ). [5]

OR

Two solid cones A and B placed in a cylindrical tube as shown in the figure. The ratio of their capacities are 2 : 1. Find the heights and capacities of cones. Also, find the volume of the remaining portion of the cylinder.



35. If the median of the distribution given below is 28.5, then find the values of x and y. [5]

Class Interval	frequency
0-10	5
10-20	x
20-30	20
30-40	15
40-50	y
50-60	5
Total	60

**Section E**

36. **Read the text carefully and answer the questions:** [4]

Suman is celebrating his birthday. He invited his friends. He bought a packet of toffees/candies which contains 360 candies. He arranges the candies such that in the first row there are 3 candies, in second there are 5 candies, in third there are 7 candies and so on.

- (i) Find the total number of rows of candies.
- (ii) How many candies are placed in last row?

OR

Find the number of candies in 12th row.

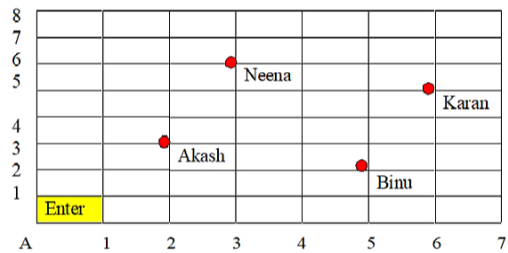
- (iii) If Aditya decides to make 15 rows, then how many total candies will be placed by him with the same arrangement?

37. **Read the text carefully and answer the questions:** [4]

Karan went to the Lab near to his home for COVID 19 test along with his family members.

The seats in the waiting area were as per the norms of distancing during this pandemic (as shown in the figure).

His family member took their seats surrounded by red circular area.



- (i) What is the distance between Neena and Karan?
- (ii) What are the coordinates of seat of Akash?

**OR**

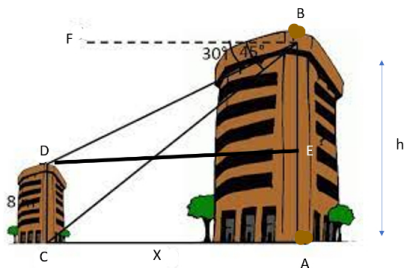
Find distance between Binu and Karan.

- (iii) What will be the coordinates of a point exactly between Akash and Binu where a person can be?

**38. Read the text carefully and answer the questions:**

[4]

Basant and Vinod lives in a housing society in Dwarka, New Delhi. There are two building in their housing society. The first building is 8 meter tall. One day, both of them were just trying to guess the height of the other multi-storeyed building. Vinod said that it might be a 45 degree angle from the bottom of our building to the top of multi-storeyed building so the height of the building and distance from our building to this multi-storeyed building will be same. Then, both of them decided to estimate it using some trigonometric tools. Let's assume that the first angles of depression of the top and bottom of an 8 m tall building from top of a multi-storeyed building are  $30^\circ$  and  $45^\circ$ , respectively.



- (i) Now help Vinod and Basant to find the height of the multistoried building.
- (ii) Also, find the distance between two buildings.

**OR**

Find the distance between top of multistoried building and bottom of first building.

- (iii) Find the distance between top of multistoried building and top of first building.

# Solution

## Section A

- (c) 2

**Explanation:** Since  $5 + 3 = 8$ , the least prime factor of  $a + b$  has to be 2, unless  $a + b$  is a prime number greater than 2. If  $a + b$  is a prime number greater than 2, then  $a + b$  must be an odd number. So, either  $a$  or  $b$  must be an even number. If  $a$  is even, then the least prime factor of  $a$  is 2, which is not 3 or 5. So, neither  $a$  nor  $b$  can be an even number. Hence,  $a + b$  cannot be a prime number greater than 2 if the least prime factor of  $a$  is 3 or 5.
- (c) irrational number

**Explanation:** Here, 3 is rational and  $2\sqrt{5}$  is irrational. We know that the sum of a rational and an irrational is an irrational number, therefore,  $3 + 2\sqrt{5}$  is irrational.
- (d)  $c = a$

**Explanation:** Product of roots =  $\frac{c}{a}$ . Also  $(\alpha \times \frac{1}{\alpha}) = 1$ .  
 $\therefore \frac{c}{a} = 1 \Rightarrow c = a$ .
- (d) 10

**Explanation:** For a system of equations  $a_1x + b_1y + c_1 = 0$ ;  $a_2x + b_2y + c_2 = 0$  to have no solution, the condition to be satisfied is  
 $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$   
 $\Rightarrow \frac{1}{5} = \frac{2}{k} \neq \frac{-3}{7}$   
 $\therefore$  For  $k = 10$ , the given system of equation is inconsistent.
- (a) 2

**Explanation:** Here,  $ax^2 + ax + 2 = 0$ ..... (1)  
 $x^2 + x + b = 0$ ..... (2)  
Putting the value of  $x = 1$  in equation (2) we get  
 $1^2 + 1 + b = 0$   
 $2 + b = 0$   
 $b = -2$   
Now, putting the value of  $x = 1$  in equation (1) we get  
 $a + a + 2 = 0$   
 $2a + 2 = 0$   
 $a = \frac{-2}{2}$   
 $= -1$   
Then,  
 $ab = (-1) \times (-2) = 2$
- (c) -1

**Explanation:** A(2, 3) and B(-4, 1) are the given points.  
Let C(0,y) be the points on y-axis  
 $AC = \sqrt{(0 - 2)^2 + (y - 3)^2}$   
 $\Rightarrow AC = \sqrt{4 + y^2 + 9 - 6y}$   
 $\Rightarrow AC = \sqrt{y^2 - 6y + 13}$   
 $BC = \sqrt{(0 + 4)^2 + (y - 1)^2}$   
 $\Rightarrow BC = \sqrt{16 + y^2 + 1 - 2y}$

$$\Rightarrow BC = \sqrt{y^2 - 2y + 17}$$

Since  $AC = BC$

$$AC^2 = BC^2$$

$$y^2 - 6y + 13 = y^2 - 2y + 17$$

$$\Rightarrow -6y + 2y = 17 - 13$$

$$\Rightarrow -4y = 4$$

$$\Rightarrow y = -1$$

Therefore, the point on y-axis is  $(0, -1)$  and here ordinate is  $-1$ .

7.

(b) 7.5 cm

**Explanation:**  $\because \triangle ABC \sim \triangle DEF$

$$\therefore \frac{\text{Perimeter}(\triangle ABC)}{\text{Perimeter}(\triangle DEF)} = \frac{AB}{DE}$$

$$\Rightarrow \frac{32}{24} = \frac{10}{DE}$$

$$\Rightarrow DE = \frac{10 \times 24}{32} = 7.5 \text{ cm}$$

8.

(d) 4 cm.

**Explanation:** Since  $XY \parallel BC$ , then using Thales theorem,

$$\Rightarrow \frac{AX}{AB} = \frac{XY}{BC}$$

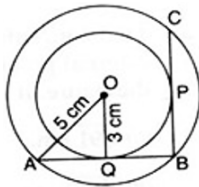
$$\Rightarrow \frac{3}{4.5} = \frac{XY}{6}$$

$$\Rightarrow XY = 4 \text{ cm}$$

9.

(b) 8 cm

**Explanation:**



Construction: Joined OP.

In right angled triangle AOQ,

$$AQ = \sqrt{(5)^2 - (3)^2} = \sqrt{25 - 9} = \sqrt{16} = 4 \text{ cm}$$

Since perpendicular from centre bisect opposite sides.

$$\therefore AQ = QB = 4 \text{ cm}$$

Also  $QB = PB = 4 \text{ cm}$  [Tangents to a circle]

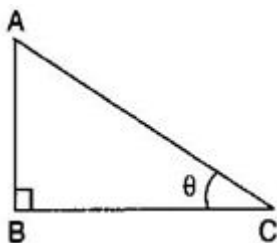
And  $PB = PC = 4 \text{ cm}$  [ $OP \perp BC$ ]

$$\therefore BC = PB + PC = 4 + 4 = 8 \text{ cm}$$

10.

(d) Base = Perpendicular

**Explanation:**



Given: in triangle ABC,  $\angle C = 45^\circ$ , and  $\angle B = 90^\circ$ ,

$$\text{Since, } \tan \theta = \frac{\text{Perpendicular}}{\text{Base}}$$

$$\Rightarrow \tan 45^\circ = \frac{\text{Perpendicular}}{\text{Base}}$$

$$\Rightarrow 1 = \frac{\text{Perpendicular}}{\text{Base}}$$

$$\Rightarrow \text{Base} = \text{perpendicular}$$

11. (a)  $30^\circ$

**Explanation:** Let AB be the tower and B be the kite.

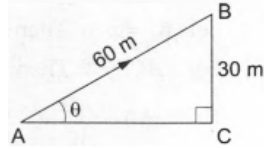


Let AC be the horizontal and let  $BC \perp AC$ .

Let  $\angle CAB = \theta$ .

$BC = 30$  m and  $AB = 60$  m. Then,

$$\frac{BC}{AB} = \sin \theta \Rightarrow \sin \theta = \frac{30}{60} = \frac{1}{2} \Rightarrow \sin \theta = \sin 30^\circ \Rightarrow \theta = 30^\circ.$$



12. (a)  $\sqrt{2}$

**Explanation:** Given:  $\sin 45^\circ + \cos 45^\circ$

$$= \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}$$

$$= \frac{2}{\sqrt{2}} = \sqrt{2}$$

13. (a)  $100^\circ$

**Explanation:** We have given that area of the sector is  $\frac{5}{18}$  of the area of the circle.

Therefore, area of the sector =  $\frac{5}{18} \times$  area of the circle

$$\Rightarrow \frac{\theta}{360} \times \pi r^2 = \frac{5}{18} \times \pi r^2$$

Now we will simplify the equation as below,

$$\Rightarrow \frac{\theta}{360} = \frac{5}{18}$$

$$\therefore \theta = \frac{5}{18} \times 360$$

$$\therefore \theta = 100$$

Therefore, sector angle is  $100^\circ$ .

14. (a)  $231 \text{ cm}^2$

**Explanation:** The angle subtended by the arc =  $60^\circ$

$$\text{So, area of the sector} = \left(\frac{60^\circ}{360^\circ}\right) \times \pi r^2 \text{ cm}^2$$

$$= \left(\frac{441}{6}\right) \times \left(\frac{22}{7}\right) \text{ cm}^2$$

$$= 231 \text{ cm}^2$$

15.

(b)  $\frac{12}{25}$

**Explanation:** Number of multiples of 3 = 8 ( 3 6 9 12 15 18 21 24 )

Number of multiples of 5 = 5 ( 5 10 15 20 25 )

Number of possible outcomes (multiples of 3 or 5) = 12 ( 3,5,6,9,10,12,15,18,20,21,24,25 )

Number of Total outcomes = 25

$$\therefore \text{Required Probability} = \frac{12}{25}$$

16.

(c) mean = mode = median

**Explanation:** For a symmetrical distribution,

we have Mean = mode = median

17.

(c)  $\frac{1}{3} \pi r^3$

**Explanation:**



$$\text{Volume of cone} = \frac{1}{3} \pi r^2 h$$

Here height of the carved out cone = Radius of the hemisphere

$$\therefore \text{Volume of cone} = \frac{1}{3} \pi r^2 \times r = \frac{1}{3} \pi r^3$$

18.

(d) 27

**Explanation:** Given that,  $u_i = \frac{x_i - 25}{10}$ ,  $\sum f_i u_i = 20$ ,  $\sum f_i = 100$

Here assumed mean = 25 and class interval (h) = 10

$$\begin{aligned} \therefore \bar{x} &= A + \frac{\sum f_i u_i}{\sum f_i} \times h = 25 + \frac{20}{100} \times 10 \\ &= 25 + 2 = 27 \end{aligned}$$

19.

**(b)** Both A and R are true but R is not the correct explanation of A.

**Explanation:** It will be  $\sqrt{(a-0)^2 + (b-0)^2} = \sqrt{a^2 + b^2}$

20.

**(d)** A is false but R is true.

**Explanation:** We know that for any two numbers, Product of the two numbers = HCF  $\times$  LCM

$$\text{HCF} \times \text{LCM} = 18 \times 169 = 3042 \neq 3072$$

So, A is false but R is true.

### Section B

21.  $5x - 4y - 8 = 0$

$$7x + 6y - 9 = 0$$

Here,  $a_1 = 5$ ,  $b_1 = -4$ ,  $c_1 = 8$

$$a_2 = 7$$
,  $b_2 = 6$ ,  $c_2 = 9$

We see that  $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

Hence, the lines representing the given pair of linear equations intersect at the point and the equations are consistent having unique solution.

22. In  $\triangle ABC$ ,  $AB \parallel DE$ .

$$\therefore \frac{CD}{DA} = \frac{CE}{EB} \dots \text{(i) [by Thales' theorem]}$$

In  $\triangle CDB$ ,  $BD \parallel EF$

$$\therefore \frac{CF}{FD} = \frac{CE}{EB} \dots \text{(ii) [by Thales' theorem]}$$

From (i) and (ii) we get

$$\frac{CD}{DA} = \frac{CF}{FD}$$

$$\Rightarrow \frac{DA}{DC} = \frac{FD}{CF} \text{ [taking reciprocals]}$$

$$\Rightarrow \frac{DA}{DC} + 1 = \frac{FD}{CF} + 1$$

$$\Rightarrow \frac{DA+DC}{DC} = \frac{FD+CF}{CF}$$

$$\Rightarrow \frac{AC}{DC} = \frac{DC}{CF}$$

$$\Rightarrow DC^2 = CF \times AC$$

OR

In triangles ABC and DBC

$$\angle A = \angle D = 90^\circ$$

AC and BD intersect each other at E

$$AE \times EC = ED \times BE$$

In triangles AEB and EDC,

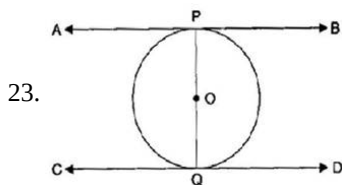
$$\angle AEB = \angle DEC \dots \text{(Vertically opposite angles)}$$

triangle ABE  $\sim$  EDC

$$(EB/EC) = (AE/DE)$$

$$EB \times DE = EC \times AE$$

Hence,  $AE \times EC = ED \times BE$



23.

Given: PQ is a diameter of a circle with centre O.

The lines AB and CD are the tangents at P and Q respectively.

To Prove:  $AB \parallel CD$

Proof: Since AB is a tangent to the circle at P and OP is the radius through the point of contact.



$$\therefore \angle OPA = 90^\circ \dots\dots (i)$$

[The tangent at any point of a circle is  $\perp$  to the radius through the point of contact]

$\therefore$  CD is a tangent to the circle at Q and OQ is the radius through the point of contact.

$$\therefore \angle OQD = 90^\circ \dots\dots (ii)$$

[The tangent at any point of a circle is  $\perp$  to the radius through the point of contact]

From eq. (i) and (ii),  $\angle OPA = \angle OQD$

But these form a pair of equal alternate angles also,

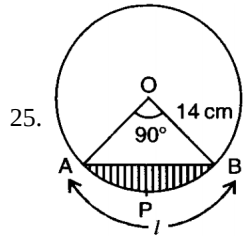
$$\therefore AB \parallel CD$$

24. Given  $\tan A = 1$  and  $\sin B = \frac{1}{\sqrt{2}}$

$$\Rightarrow \tan A = \tan 45^\circ \text{ and } \sin B = \sin 45^\circ$$

$$\Rightarrow A = 45^\circ \text{ and } B = 45^\circ$$

$$\text{Now } \cos(A+B) = \cos(45^\circ + 45^\circ) = \cos 90^\circ = 0.$$



$$l = APB = 22 \text{ cm}$$

$$\frac{\theta}{180^\circ} \times \frac{22}{7} \times 14 = 22 \text{ cm}$$

$$\Rightarrow \theta = 90^\circ$$

$$\text{Area of the sector} = \frac{lr}{2} = \frac{22 \times 14}{2} = 154 \text{ cm}^2$$

$$\text{Area of triangle } AOB = \frac{1}{2} \times OA \times OB = \frac{1}{2} \times 14 \times 14 = 98 \text{ cm}^2$$

$$\text{Area of the segment} = (154 - 98) \text{ cm}^2 = 56 \text{ cm}^2$$

OR



$$\text{Given Radius} = r = 5\sqrt{2} \text{ cm}$$

$$= OA = OB$$

$$\text{Length of chord } AB = 10 \text{ cm}$$

$$\text{In } \triangle OAB, OA = OB = 5\sqrt{2}$$

$$AB = 10 \text{ cm}$$

$$OA^2 + OB^2 = (5\sqrt{2})^2 + (5\sqrt{2})^2$$

$$= 50 + 50 = 100 = (AB)^2$$

Pythagoras theorem is satisfied OAB is right triangle

$$= \text{angle subtended by chord} = \angle AOB = 90^\circ$$

Area of segment (minor) = shaded region

$$= \text{area of sector} - \text{area of } \triangle OAB$$

$$= \frac{\theta}{360} \times \pi r^2 - \frac{1}{2} \times OA \times OB$$

$$= \frac{90}{360} \times \frac{22}{7} (5\sqrt{2})^2 - \frac{1}{2} \times 5\sqrt{2} \times 5\sqrt{2}$$

$$= \frac{275}{7} - 25 = \frac{100}{7} \text{ cm}^2$$

Area of major segment = (area of circle) - (area of minor segment)

$$= \pi r^2 - \frac{100}{7}$$

$$= \frac{22}{7} \times (5\sqrt{2})^2 - \frac{100}{7}$$

$$= \frac{1100}{7} - \frac{100}{7}$$

$$= \frac{1000}{7} \text{ cm}^2$$

Section C

26. Let us assume, to the contrary, that  $5 - \sqrt{3}$  is rational.

That is, we can find coprime numbers  $a$  and  $b$  ( $b \neq 0$ ) such that  $5 - \sqrt{3} = \frac{a}{b}$

Therefore,  $5 - \frac{a}{b} = \sqrt{3}$

Rearranging this equation, we get  $\sqrt{3} = 5 - \frac{a}{b} = \frac{5b-a}{b}$

Since  $a$  and  $b$  are integers, we get  $5 - \frac{a}{b}$  is rational, and so  $\sqrt{3}$  is rational.

But this contradicts the fact that  $\sqrt{3}$  is irrational

This contradiction has arisen because of our incorrect assumption that  $5 - \sqrt{3}$  is rational.

So, we conclude that  $5 - \sqrt{3}$  is irrational.

27.  $f(x) = x^2 - 2x - 8$

$= x^2 - 4x + 2x - 8$

$= x(x - 4) + 2(x - 4)$

$= (x + 2)(x - 4)$

$f(x) = 0$  if  $x+2 = 0$  or  $x-4 = 0$

$x = -2$  or  $4$

So the zeroes of the polynomials are  $-2$  and  $4$ .

For the Polynomial  $f(x)=x^2 - 2x - 8$

$a=1, b=-2, c=-8$

Sum of the zeroes  $= -2 + 4 = 2 = -\frac{b}{a}$

Product of zeros  $= (-2)(4) = -8 = \frac{c}{a}$

Hence, the relationship between the zeros and coefficients is verified.

28. Let the unit's digit and the ten's digit in the two-digit number be  $x$  and  $y$  respectively.

Then the number  $= 10y + x$

Also, the number obtained by reversing the order of the digits  $= 10x + y$

According to the question,

$x + y = 9$ .....(1)

$9(10y + x) = 2(10x + y)$

$\Rightarrow 90y + 9x = 20x + 2y$

$\Rightarrow 11x - 88y = 0$

$\Rightarrow x - 8y = 0$  .....(2)

Subtracting equation(2) from equation(1), we get

$9y = 9$

$\Rightarrow y = \frac{9}{9} = 1$

Substituting this value of  $y$  in equation (1), we get

$x + 1 = 9$

$\Rightarrow x = 9 - 1 = 8$

Hence, the required number is  $18$ .

**Verification:** substituting  $x = 8$  and  $y = 1$ ,

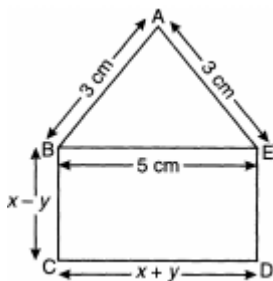
we find that both the equations (1) and (2) are satisfied as shown below:

$x + y = 8 + 1 = 9$

$x - 8y = 8 - 8(1) = 0$

Hence, the solution is correct.

OR



Since  $BC \parallel DE$  and  $BE \parallel CD$  with  $BC \perp CD$ ,  $BCDE$  is a rectangle.

Since,  $BE = CD$

$\therefore x + y = 5$  ..(i)

Also,  $DE = BC = x - y$

Since, perimeter of ABCDE is 21

$$\therefore AB + BC + CD + DE + EA = 21$$

$$\Rightarrow 3 + x - y + x + y + x - y + 3 = 21$$

$$\Rightarrow 6 + 3x - y = 21$$

$$\Rightarrow 3x - y = 15 \dots (ii)$$

Adding eqns. (i) and (ii), we get

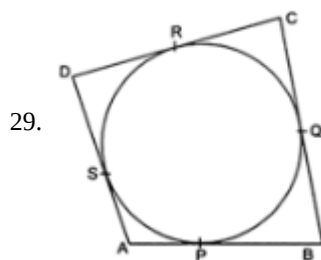
$$4x = 20$$

$$\Rightarrow x = 5$$

On substituting the value of x in (i), we get

$$y = 0$$

$$\therefore x = 5 \text{ and } y = 0.$$



It is given that ABCD is the quadrilateral circumscribing the circle.

Let the quadrilateral touches the circle at points P, Q, R and S.

As we know that length of tangents drawn from an external point are always equal),

therefore,  $AP = AS \dots \dots \dots (i)$

$BP = BQ \dots \dots \dots (ii)$

$CR = CQ \dots \dots \dots (iii)$

$DR = DS \dots \dots \dots (iv)$

Adding (i) + (ii) + (iii) + (iv), we obtain

$$AP + BP + CR + DR = AS + BQ + CQ + DS$$

$$(AP + BP) + (CR + DR) = (AS + DS) + (BQ + CQ)$$

$$AB + CD = AD + BC$$

Hence proved

30. We have to prove that,  $\frac{\sin \theta - \cos \theta + 1}{\sin \theta + \cos \theta - 1} = \frac{1}{\sec \theta - \tan \theta}$  using identity  $\sec^2 \theta = 1 + \tan^2 \theta$

$$\begin{aligned} \text{LHS} &= \frac{\sin \theta - \cos \theta + 1}{\sin \theta + \cos \theta - 1} = \frac{\tan \theta - 1 + \sec \theta}{\tan \theta + 1 - \sec \theta} \text{ [ dividing the numerator and denominator by } \cos \theta \text{.]} \\ &= \frac{(\tan \theta + \sec \theta) - 1}{(\tan \theta - \sec \theta) + 1} = \frac{\{(\tan \theta + \sec \theta) - 1\}(\tan \theta - \sec \theta)}{\{(\tan \theta - \sec \theta) + 1\}(\tan \theta - \sec \theta)} \text{ [ Multiplying and dividing by } (\tan \theta - \sec \theta) \text{]} \\ &= \frac{(\tan^2 \theta - \sec^2 \theta) - (\tan \theta - \sec \theta)}{\{(\tan \theta - \sec \theta) + 1\}(\tan \theta - \sec \theta)} \text{ [ } \because (a - b)(a + b) = a^2 - b^2 \text{ ]} \\ &= \frac{-1 - \tan \theta + \sec \theta}{(\tan \theta - \sec \theta + 1)(\tan \theta - \sec \theta)} \text{ [ } \because \tan^2 \theta - \sec^2 \theta = -1 \text{ ]} \\ &= \frac{-1 - \tan \theta + \sec \theta}{-(\tan \theta - \sec \theta + 1)} = \frac{-1}{(\tan \theta - \sec \theta + 1)(\tan \theta - \sec \theta)} = \frac{-1}{\tan \theta - \sec \theta} \\ &= \frac{1}{\sec \theta - \tan \theta} = \text{RHS} \end{aligned}$$

Hence Proved.

OR

Given that,  $\sin \theta + \cos \theta = \sqrt{2}$

On squaring both the sides, we get

$$(\sin \theta + \cos \theta)^2 = (\sqrt{2})^2$$

$$\sin^2 \theta + \cos^2 \theta + 2 \sin \theta \cos \theta = 2 \text{ [ } \because (a+b)^2 = a^2 + 2ab + b^2 \text{ ]}$$

$$\Rightarrow 1 + 2 \sin \theta \cos \theta = 2$$

$$\Rightarrow 2 \sin \theta \cos \theta = 2 - 1 = 1$$

$$\Rightarrow \frac{1}{\sin \theta \cos \theta} = 2 \dots \dots \dots (i)$$

$$\text{Now, } \tan \theta + \cot \theta = \frac{\sin \theta}{\cos \theta} + \frac{\cos \theta}{\sin \theta}$$

$$\frac{\sin^2 \theta + \cos^2 \theta}{\cos \theta \sin \theta} = \frac{1}{\cos \theta \sin \theta} \dots \dots \dots (ii)$$

From (i) and (ii) we get

$$\tan \theta + \cot \theta = 2$$

31. No. of days in a leap = 366

$$\text{(i.e. } \frac{366}{7} = 52 \text{ weeks} + 2 \text{ days)}$$

So, there will be 52 weeks and 2 days

So, every leap year has 52 Sundays

Now, the probability depends on the remaining 2 days

The possible pairing of days are:

Sunday-Monday

Monday-Tuesday

Tuesday-Wednesday

Wednesday-Thursday

Thursday-Friday

Friday-Saturday

Saturday-Sunday

There are total of 7 pairs and out of 7 pairs, only 2 pairs have Sunday.

The remaining 5 pairs do not include Sunday.

$$\text{hence, the probability of not getting Sunday in the last 2 days} = \frac{5}{7}$$

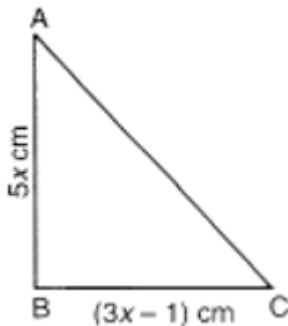
$$\text{Therefore, the probability of only 52 Sundays in a Leap year} = \frac{5}{7}$$

#### Section D

32. Area of triangle =  $\frac{1}{2} \times \text{base} \times \text{height}$

$$= \frac{1}{2} \times 5x \times (3x - 1)$$

According to the question,



$$15x^2 - 5x = 120$$

$$\text{or, } 3x^2 - x - 24 = 0$$

$$\text{or, } 3x^2 - 9x + 8x - 24 = 0$$

$$\text{or, } 3x(x - 3) + 8(x - 3) = 0$$

$$\text{or, } (x - 3)(3x + 8) = 0$$

$$\therefore x = 3, x = -\frac{8}{3}$$

Length can't be negative, so  $x = 3$

$$AB = 5 \times 3 = 15 \text{ cm, } BC = 3x - 1 = 9 - 1$$

$$= 8 \text{ cm}$$

$$AC = \sqrt{15^2 + 8^2}$$

$$= \sqrt{225 + 64}$$

$$= \sqrt{289} = 17 \text{ cm}$$

Hence hypotenuse = 17 cm

OR

Let the number of toys produced be  $x$ .

$$\therefore \text{Cost of production of each toy} = \text{Rs } (55 - x)$$

It is given that, total production of the toys = Rs 750

$$\therefore x(55 - x) = 750$$

$$\Rightarrow x^2 - 55x + 750 = 0$$

Now to factorize this equation we have to find two numbers such that their product is 750 and sum is 55

$$\Rightarrow x^2 - 25x - 30x + 750 = 0$$

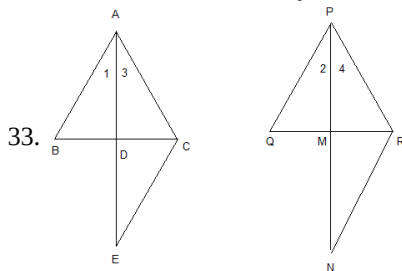
$$\Rightarrow x(x - 25) - 30(x - 25) = 0$$

$$\Rightarrow (x - 25)(x - 30) = 0$$

Either  $x - 25 = 0$  or  $x - 30 = 0$

i.e.,  $x = 25$  or  $x = 30$

Hence, the number of toys will be either 25 or 30.



Given : In  $\triangle ABC$  and  $\triangle PQR$  The AD and PM are their medians,

$$\text{such that } \frac{AB}{PQ} = \frac{AD}{PM} = \frac{AC}{PR}$$

To prove :  $\triangle ABC \sim \triangle PQR$

Construction : Produce AD to E such that  $AD = DE$  and produce PM to N such that  $PM = MN$ . Join CE and RN.

Proof : In  $\triangle ABD$  and  $\triangle EDC$

$$AD = DE$$

$$\angle ADB = \angle EDC \text{ (vertically opposite angles)}$$

$$BD = DC \text{ (as AD is a median)}$$

$$\therefore \triangle ABD \cong \triangle EDC \text{ (By SAS congruency)}$$

$$\text{or, } AB = CE \text{ (By CPCT)}$$

Similarly,  $PQ = RN$

$$\frac{AB}{PQ} = \frac{AD}{PM} = \frac{AC}{PR} \text{ (Given)}$$

$$\text{or, } \frac{CE}{RN} = \frac{2AD}{2PM} = \frac{AC}{PR}$$

$$\text{or } \frac{CE}{RN} = \frac{AE}{PN} = \frac{AC}{PR}$$

So  $\triangle ACE \sim \triangle PRN$

$$\angle 3 = \angle 4$$

$$\text{Similarly } \angle 1 = \angle 2$$

$$\angle 1 + \angle 3 = \angle 2 + \angle 4$$

So  $\angle A = \angle P$  and

$$\frac{AB}{PQ} = \frac{AC}{PR} \text{ (given)}$$

Hence  $\triangle ABC \sim \triangle PQR$

34. According to question it is given that

Diameter of the base of the cone is = 7cm

Therefore radius =  $\frac{7}{2} = 3.5$ cm

Total height of the toy = 14.5 cm

Height of the cone =  $15.5 - 3.5 = 12$  cm

Height of the hemisphere = 3.5 cm

According to question it is also given that

Volume of the toy = Volume of cone + Volume of hemisphere

$$= \frac{1}{3}\pi r^2 h + \frac{2}{3}\pi r^2$$

$$= \frac{1}{3}\pi r^2 (2r + h)$$

$$= \frac{1}{3} \times \frac{22}{7} \times (3.5)^2 [2 \times 3.5 + 12]$$

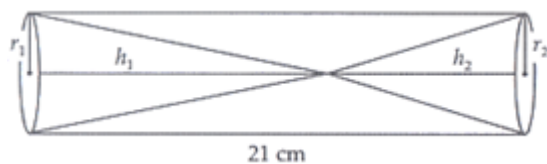
$$= \frac{1}{3} \times 22 \times 1.75 \times 19$$

$$= 243.83 \text{ cm}^3$$

OR

Let height of the cone 1 be 'h' cm and the height of the cone 2 be  $(21 \text{ cm} - h)$ .

As the ratio of volumes of cone  $c_1$  and  $c_2$  is 2 : 1, their radii are same equal to  $r = \frac{6}{2} \text{ cm} = 3 \text{ cm}$ .



$$\therefore \frac{V_1}{V_2} = \frac{\frac{1}{3}\pi r_1^2 h_1}{\frac{1}{3}\pi r_2^2 h_2}$$

$$\Rightarrow \frac{2}{1} = \frac{h}{21cm-h}$$

$$\text{or } 42 \text{ cm} - 2h = h$$

$$\text{or, } 3h = 42 \text{ cm}$$

$$\Rightarrow h = 42/3$$

$$\Rightarrow h = 14 \text{ cm}$$

Hence, height of cone 1 = 14 cm and height of cone 2 = 7 cm

Cone I	Cone II	Cylinder
$r_1 = \frac{6}{3} = 3 \text{ cm}$	$r_2 = 3 \text{ cm}$	$r = 3 \text{ cm}$
$h_1 = 14 \text{ cm}$	$h_2 = 7 \text{ cm}$	$h = 21 \text{ cm}$

$$\text{Volume of cone 1} = \frac{1}{3}\pi r_1^2 h_1 = \frac{1}{3} \times \frac{22}{7} \times 3 \times 3 \times 14 = 132 \text{ cm}^3$$

$$\text{Volume of cone 2} = \frac{1}{3}\pi r_2^2 h_2 = \frac{1}{3} \times \frac{22}{7} \times 3 \times 3 \times 7 = 22 \times 3 = 66 \text{ cm}^3$$

Volume of remaining portion of tube = Vol. of cylinder – Vol. of cone 1 – Vol. of cone 2

$$= \pi r^2 h - 132 - 66$$

$$= \frac{22}{7} \times 3 \times 3 \times 21 - 198$$

$$= 22 \times 27 - 198 = 594 - 198 = 396 \text{ cm}^3$$

Hence, the required volume is  $396 \text{ cm}^3$ .

35.

Monthly Consumption	Number of consumers ( $f_i$ )	Cumulative Frequency
0-10	5	5
10-20	x	5 + x
20-30	20	25 + x
30-40	15	40 + x
40-50	y	40 + x + y
50-60	5	45 + x + y
Total	$\sum f_i = n = 60$	

Here,  $\sum f_i = n = 60$ , then  $\frac{n}{2} = \frac{60}{2} = 30$ , also, median of the distribution is 28.5, which lies in interval 20 – 30.

$\therefore$  Median class = 20 – 30

So,  $l = 20$ ,  $n = 60$ ,  $f = 20$ ,  $cf = 5 + x$  and  $h = 10$

$$\therefore 45 + x + y = 60$$

$$\Rightarrow x + y = 15 \dots\dots\dots(i)$$

$$\text{Now, Median} = l + \left[ \frac{\frac{n}{2} - cf}{f} \right] \times h$$

$$\Rightarrow 28.5 = 20 + \left[ \frac{30 - (5+x)}{20} \right] \times 10$$

$$\Rightarrow 28.5 = 20 + \frac{30 - 5 - x}{2}$$

$$\Rightarrow 28.5 = \frac{40 + 25 - x}{2}$$

$$\Rightarrow 57.0 = 65 - x$$

$$\Rightarrow x = 65 - 57 = 8$$

$$\Rightarrow x = 8$$

Putting the value of x in eq. (i), we get,

$$8 + y = 15$$

$$\Rightarrow y = 7$$

Hence the value of x and y are 8 and 7 respectively.

**Section E**

36. Read the text carefully and answer the questions:



Suman is celebrating his birthday. He invited his friends. He bought a packet of toffees/candies which contains 360 candies. He arranges the candies such that in the first row there are 3 candies, in second there are 5 candies, in third there are 7 candies and so on.

(i) Let there be 'n' number of rows

Given 3, 5, 7... are in AP

First term  $a = 3$  and common difference  $d = 2$

$$S_n = \frac{n}{2}[2a + (n - 1)d]$$

$$\Rightarrow 360 = \frac{n}{2}[2 \times 3 + (n - 1) \times 2]$$

$$\Rightarrow 360 = n[3 + (n - 1) \times 1]$$

$$\Rightarrow n^2 + 2n - 360 = 0$$

$$\Rightarrow (n + 20)(n - 18) = 0$$

$$\Rightarrow n = -20 \text{ reject}$$

$$n = 18 \text{ accept}$$

(ii) Since there are 18 rows number of candies placed in last row (18<sup>th</sup> row) is

$$a_n = a + (n - 1)d$$

$$\Rightarrow a_{18} = 3 + (18 - 1)2$$

$$\Rightarrow a_{18} = 3 + 17 \times 2$$

$$\Rightarrow a_{18} = 37$$

OR

The number of candies in 12th row.

$$a_n = a + (n - 1)d$$

$$\Rightarrow a_{12} = 3 + (12 - 1)2$$

$$\Rightarrow a_{12} = 3 + 11 \times 2$$

$$\Rightarrow a_{12} = 25$$

(iii) If there are 15 rows with same arrangement

$$S_n = \frac{n}{2}[2a + (n - 1)d]$$

$$\Rightarrow S_{15} = \frac{15}{2}[2 \times 3 + (15 - 1) \times 2]$$

$$\Rightarrow S_{15} = 15[3 + 14 \times 1]$$

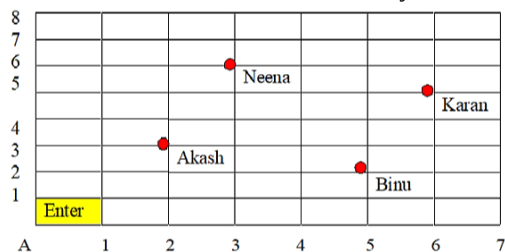
$$\Rightarrow S_{15} = 255$$

There are 255 candies in 15 rows.

**37. Read the text carefully and answer the questions:**

Karan went to the Lab near to his home for COVID 19 test along with his family members.

The seats in the waiting area were as per the norms of distancing during this pandemic (as shown in the figure). His family member took their seats surrounded by red circular area.



(i) Position of Neena = (3, 6)

Position of Karan = (6, 5)

$$\text{Distance between Neena and Karan} = \sqrt{(6 - 3)^2 + (5 - 6)^2}$$

$$= \sqrt{9 + (-1)^2}$$

$$= \sqrt{10}$$

(ii) Co-ordinate of seat of Akash = 2, 3

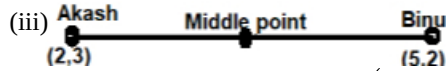
OR

Binu = (5, 5); Karan = (6, 5)

$$\text{Distance} = \sqrt{(6 - 5)^2 + (5 - 2)^2}$$

$$= \sqrt{1+9}$$

$$= \sqrt{10}$$

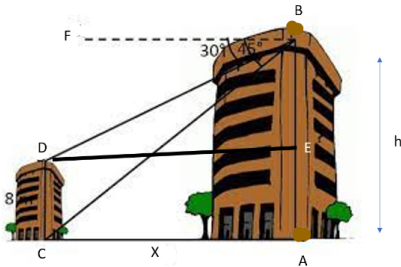


$$\text{Co-ordinate of middle point} = \left( \frac{2+5}{2}, \frac{3+2}{2} \right)$$

$$= 3.5, 2.5$$

**38. Read the text carefully and answer the questions:**

Basant and Vinod lives in a housing society in Dwarka, New Delhi. There are two building in their housing society. The first building is 8 meter tall. One day, both of them were just trying to guess the height of the other multi-storeyed building. Vinod said that it might be a 45 degree angle from the bottom of our building to the top of multi-storeyed building so the height of the building and distance from our building to this multi-storeyed building will be same. Then, both of them decided to estimate it using some trigonometric tools. Let's assume that the first angles of depression of the top and bottom of an 8 m tall building from top of a multi-storeyed building are 30° and 45°, respectively.



(i) Let h is height of big building, here as per the diagram.

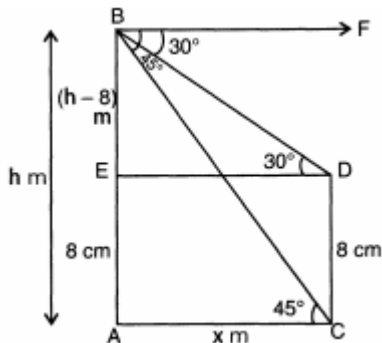
$$AE = CD = 8 \text{ m (Given)}$$

$$BE = AB - AE = (h - 8) \text{ m}$$

$$\text{Let } AC = DE = x$$

$$\text{Also, } \angle FBD = \angle BDE = 30^\circ$$

$$\angle FBC = \angle BCA = 45^\circ$$



In  $\triangle ACB$ ,  $\angle A = 90^\circ$

$$\tan 45^\circ = \frac{AB}{AC}$$

$$\Rightarrow x = h, \dots(i)$$

In  $\triangle BDE$ ,  $\angle E = 90^\circ$

$$\tan 30^\circ = \frac{BE}{ED}$$

$$\Rightarrow x = \sqrt{3}(h - 8) \dots(ii)$$

From (i) and (ii), we get

$$h = \sqrt{3}h - 8\sqrt{3}$$

$$h(\sqrt{3} - 1) = 8\sqrt{3}$$

$$h = \frac{8\sqrt{3}}{\sqrt{3}-1} = \frac{8\sqrt{3}}{\sqrt{3}-1} \times \frac{\sqrt{3}+1}{\sqrt{3}+1}$$

$$= \frac{1}{2} \times (24 + 8\sqrt{3}) = \frac{1}{2} \times (24 + 13.84) = 18.92 \text{ m}$$

(ii) Let h is height of big building, here as per the diagram.

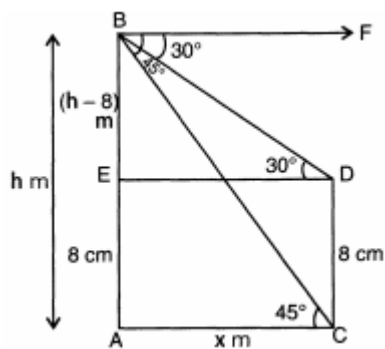
$$AE = CD = 8 \text{ m (Given)}$$

$$BE = AB - AE = (h - 8) \text{ m}$$

$$\text{Let } AC = DE = x$$

$$\text{Also, } \angle FBD = \angle BDE = 30^\circ$$

$$\angle FBC = \angle BCA = 45^\circ$$



In  $\triangle ACB$ ,  $\angle A = 90^\circ$

$$\tan 45^\circ = \frac{AB}{AC}$$

$$\Rightarrow x = h, \dots(i)$$

In  $\triangle BDE$ ,  $\angle E = 90^\circ$

$$\tan 30^\circ = \frac{BE}{ED}$$

$$\Rightarrow x = \sqrt{3}(h - 8) \dots(ii)$$

From (i) and (ii), we get

$$h = \sqrt{3}h - 8\sqrt{3}$$

$$h(\sqrt{3} - 1) = 8\sqrt{3}$$

$$h = \frac{8\sqrt{3}}{\sqrt{3}-1} = \frac{8\sqrt{3}}{\sqrt{3}-1} \times \frac{\sqrt{3}+1}{\sqrt{3}+1}$$

$$= \frac{1}{2} \times (24 + 8\sqrt{3}) = \frac{1}{2} \times (24 + 13.84) = 18.92 \text{ m}$$

Hence height of the multistory building is 18.92 m and the distance between two buildings is 18.92 m.

OR

In  $\triangle ABC$

$$\sin 45^\circ = \frac{AB}{BC}$$

$$\Rightarrow BC = \frac{AB}{\sin 45^\circ}$$

$$\Rightarrow BC = \frac{18.92}{\frac{1}{\sqrt{2}}}$$

$$\Rightarrow BC = 26.76 \text{ m}$$

Hence the distance between top of multistoried building and bottom of first building is 26.76 m.

(iii) In  $\triangle BDE$

$$\cos 30^\circ = \frac{ED}{BD}$$

$$\Rightarrow BD = \frac{ED}{\cos 30^\circ}$$

$$\Rightarrow BD = \frac{8\sqrt{3}}{\frac{\sqrt{3}-1}{2}} = \frac{16}{\sqrt{3}-1}$$

$$\Rightarrow BD = 8(\sqrt{3} + 1) = 21.86 \text{ m}$$

Hence, the distance between top of multistoried building and top of first building is 21.86 m.